# PROBABILITY DISTRIBUTION

Imagine you have a bag of differently colored marbles, and you want to understand how likely it is to pick each color. A probability distribution is like a chart or a set of rules that tells you the chances of picking each color.

If all the marbles are equally likely to be picked, you have a **uniform distribution**. Each color has the same chance.

If some colors are more common, you might have a distribution where certain colors have higher probabilities of being picked. For instance, you're more likely to pick red marbles than blue marbles.

On the other hand, some distributions might make it very unlikely to pick certain colors. Maybe there's a rare, special marble that's hard to get.

Now, think about this in terms of everyday situations:

* In weather forecasting, a probability distribution could describe the chances of different weather conditions (sunny, rainy, cloudy) on a given day.
* In finance, it could tell you the probability of making different amounts of money when investing in stocks.
* In a classroom, it might show the likelihood of getting different scores on a test.

In essence, a probability distribution is a **tool to understand and quantify uncertainty or randomness**. It helps us make informed decisions by knowing the odds of different outcomes in various situations.

# DEFINITION

A probability distribution is a mathematical function that specifies the likelihood of different outcomes or events in a random experiment.

Probability distributions can be categorized into two primary groups based on the type of data they are intended to model:

1. **Discrete Distributions**: These distributions are used to model discrete data, where the random variable can only take on specific, distinct values with gaps in between. These values are typically finite and countable. Discrete distributions are often associated with situations involving counts, integers, or events that are distinct and separate from each other. Common examples include the Poisson distribution, binomial distribution, and geometric distribution. **A Probability Mass Function is used for discrete random variables.** Discrete random variables take on a countable number of distinct values, and the PMF assigns a probability to each of these values.
2. **Continuous Distributions**: Continuous distributions are designed for continuous data, where the random variable can take on an infinite number of possible values within a given range. These distributions are used when data can vary smoothly and can include any real number within an interval. Continuous distributions are characterized by probability density functions and are often used to model measurements or observations that are not restricted to specific values. The normal (Gaussian) distribution, exponential distribution, and uniform distribution are examples of continuous distributions. **A Probability Density Function is used for continuous random variables**. Continuous random variables can take on an uncountable number of values within a range. Unlike the PMF, which assigns probabilities to specific values, the PDF assigns probabilities to intervals of values.

# DESCRETE PROBABILITY DISTRIBUTION

## BINOMIAL DISTRIBUTION

It models the probability of getting a certain number of successes (usually denoted as **r**) in a series of experiments or trials, where each trial has only two possible outcomes: success or failure. The term **binomial** comes from the fact that there are two possible outcomes.

The formula for finding **binomial probability** is given by:

A close up of a letter

Description automatically generated

Where **n** is **the number of trials**, **p** is the **probability of success**, and **r** is the **number of successes after n trials**.

However, there are some **conditions** that need to be met in order for us to be able to apply the formula.

1. The **total number** of trials is **fixed** at **n**.
2. Each trial is **binary**, i.e., it has **only two possible outcomes:** success or failure.
3. **Probability of success** is the **same** in all trials, denoted by **p**.

## POISSON DISTRIBUTION

The Poisson distribution is a probability distribution that models the number of rare events occurring within a fixed time or space interval. It is used when you have a known average rate of occurrence, but the exact timing or count of events is uncertain.

The probability mass function (PMF) of the Poisson distribution is given by:

A math equation with a number of symbols

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Key characteristics of a Poisson distribution include:

1. **Count of Rare Events:** It is used to model the count of rare events within a fixed time or space interval. These events are often rare, random, and independent of each other.
2. **Fixed Interval:** The distribution is defined for a fixed interval of time or space. For example, the number of phone calls received at a customer service center in an hour or the number of accidents at a specific intersection in a day.
3. **Constant Rate:** The average rate of events occurring within the interval is denoted as "λ" (lambda) and remains constant. Lambda represents the expected number of events in the given interval.

Example:

Suppose you manage a small coffee shop, and on average, you get 5 customers arriving every 15 minutes. You want to know the probability of getting exactly 7 customers in the next 15 minutes.

In this case, we have the following information:

* λ (average rate of customer arrivals) = 5 customers per 15 minutes.
* K = 7 (7 is rare or less likely since average is 5)

We want to find P(X=7), which is the probability of having 7 customers arrive in the next 15 minutes using the Poisson distribution formula:

A number and a equal sign

Description automatically generated

## GEOMETRIC DISTRIBUTION

The geometric distribution is a probability distribution that models the number of trials needed to achieve the first success in a series of independent trials. In other words, it helps us answer questions like "How many attempts does it take to succeed for the first time?"

Key Characteristics:

1. **Success on the nth Trial:** The geometric distribution focuses on the number of trials (usually denoted as "n") required to achieve the first success.
2. **Independent Trials:** Each trial is independent of the others, and the probability of success remains constant from trial to trial.
3. **Two Outcomes:** Each trial has only two possible outcomes: success or failure.

The Probability Mass Function (PMF) of the geometric distribution is given by:



Example:

Let's say you work in sales, and the probability of making a successful sales call to a potential client is 0.2 (20% chance of success). You want to know how many calls you'll need to make before achieving your first successful sale.

In this scenario:

* *p* (probability of a successful call) = 0.2.

We want to determine *P*(*X*=*n*), the probability of achieving your first successful sale on the nth sales call using the geometric distribution formula:

*P*(*X*=*n*)=(1−*p*)^(*n*−1) x *p*

Now, let's calculate it step by step:

1. Calculate 1−*p*: 1−0.2=0.81
2. Calculate (0.8) ^(*n*−1) for the first *n*−1 failed calls.
3. Multiply the result by *p* to account for the successful call.

# CONTINOUS PROBABILITY DISTRIBUTION

## UNIFORM DISTRIBUTION

The uniform distribution, also known as the rectangular distribution, is a simple and fundamental probability distribution that assigns equal probability to all values within a specified range. It's characterized by its constant probability density within that range.

## NORMAL (GAUSSIAN) DISTRIBUTION

The normal distribution, also known as the Gaussian distribution, is one of the most important and widely used probability distributions in statistics. It is characterized by its bell-shaped curve and is often used to model a wide range of natural phenomena, including measurements, errors, and various real-world data.

Key Characteristics:

1. **Symmetry:** The normal distribution is symmetric, with the mean (average) at the center of the curve. The mean, median, and mode are all equal and located at the peak of the curve.
2. **Bell-Shaped Curve:** The curve is bell-shaped and symmetrically tapers off as you move away from the mean. This shape is a result of the probability being higher for values close to the mean and decreasing as you move further away in either direction.
3. **68-95-99.7 Rule:** In a normal distribution:
   * Approximately 68% of the data falls within one standard deviation of the mean.
   * About 95% falls within two standard deviations.
   * Nearly 99.7% falls within three standard deviations.